Chaos, and what it can reveal

A conference on the occasion of Petr Šeba's 60th birthday
Hradec Králové, Czech Republic
May 9–11 2017

Programme
Abstracts
Participants

Book of Abstracts

Auspices

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Schedule

Tuesday, May 9

12:00–14:00	Registration
14:00–14:15	UHK representatives: Conference opening
	Plenary session – AULA S1 (chair: P. Exner)
14:15–15:00	P1 Y. Avron: The 2016 Physics Nobel prize: David Thouless and topological invariant
15:05–15:50	P2 T. Cheon: Dynamical systems theory of political opinion
15:55–16:20	Coffee break
	Contributed session – AULA S1 (chair: Y. Avron)
16:20–16:50	C1 F. Studnička: Chaos in the (feline) brain
16:55–17:25	C2 J. Šlégr: Ionospehere as chaotic billiard and what it can reveal
17:30–18:00	C3 R. Kříž: Chaotic behaviour of the electricity price series
18:00–20:00	Dinner break
20:00	Concert

Wednesday, May 10

	Plenary session – AULA S1 (chair: M. Kuś)
9:00–9:45	P3 P. Exner: Some unusual spectra of periodic quantum graphs
9:50–10:35	P4 A. Sadreev: Tuning of Fano resonances by rotation of continuum: wave faucet
10:35–11:00	Coffee break
	Plenary session – AULA S1 (chair: H.-J. Stöckmann)
11:00–11:45	P5 K. Życzkowski: Quantum chaos in composite systems
11:50–12:35	P6 M. Kuś: Quantum SU(3) systems: (non)integrability, quantum chaos, classical limit(s), and experiments
12:35–14:00	Lunch break
	Contributed session – AULA S1 (chair: K. Życzkowski)
14:00–14:30	C4 H. Schanz: Networks as Coherent Perfect Absorbers
14:35–15:05	C5 M. Tater: Spectral and resonance properties of the Smilansky model
15:10–15:40	C6 J. Lipovský: Fermi's rule and high-energy asymptotics for quantum graphs
15:40–16:10	Coffee break
	Contributed session – AULA S1 (chair: L. Sirko)
16:10–16:40	C7 M. Znojil: Heisenberg picture in non-Hermitian quantum mechanics
16:45–17:15	C8 V. Lotoreichik: Optimization of the lowest eigenvalue for surface δ-interactions
17:20–17:50	C9 D. Barseghyan: Semiclassical bounds on Magnetic Laplacian
17:55–18:25	C10 A. Pérez–Obiol: On the bound states of magnetic Laplacians on wedges
19:30	Banquet

Thursday, May 11

Plenary session – AULA S1 (chair: A. Sadreev)	
9:00–9:45	P7 H.-J. Stöckmann: A microwave realization of the Gaussian symplectic ensemble
9:50–10:35	P8 L. Sirko: Missing level statistics of microwave networks with violated time reversal invariance
10:35–11:00	Coffee break
Contributed session – AULA S1 (chair: T. Cheon)	
11:00–11:30	C11 D. Krejčiřík: The Brownian traveller on manifolds
11:35–12:05	C12 A. Shudo: Amphibious Complex Orbits and Dynamical Tunneling
12:10–12:40	C13 A. Sergyeyev: Integrable partial differential systems in four independent variables related to contact geometry
12:40–12:55	Closing

Plenary Lectures

The 2016 Physics Nobel prize: David Thouless and topological invariant.

Y. Avron
Technion – Israel Institute of Technology, Haifa, Israel

The 2016 Physics Nobel prize was awarded to Thouless and Kosterlitz for the discovery of a new kind of phase transition and to Thouless and Haldane for the discovery of the topological meaning of the Quantum Hall conductance.

I shall describe Thouless original finding that the Hall conductance is related to a Chern numbers and some of the beautiful mathematical physics that grew from it. No background in condensed matter physics or topology will be assumed. The talk will be elementary.

Dynamical systems theory of public opinion

T. Cheon
Kochi University of Technology, Kochi, Japan

A dynamical theory of temporal evolution of political opinions is developed in the form of extended Polya urn. Intriguing spatio-temporal patterns are found that mimics the results of real-world democratic voting. The theory offers a unifying view over preceding approaches advanced by Galam and by Mori and Hisakado.

Some unusual spectra of periodic quantum graphs

P. Exner
Doppler Institute for Mathematical Physics and Applied Mathematics, Prague, Czechia

The topic of quantum graphs which we opened with Petr thirty years ago appeared to be unexpectedly fruitful. In this talk I am going to discuss spectra of periodic quantum graphs using two simple examples, a chain of loops and a rectangular lattice. It is generally accepted that, with the exception of 'Dirichlet eigemvalues', the spectrum consists of absolutely continuous bands and that local perturbations can give rise to eigenvalues in the gaps. My goal is to show that one can observe also other types of spectral behaviour: (i) a chain in a homogeneous magnetic field can have no absolutely continuous spectrum at all, (ii) a chain in a linear magnetic can have a spectrum of a fractal nature, and (iii) even without any external field a lattice can have a finite number of spectral gaps in analogy with Bethe-Sommerfeld behaviour of the 'usual' Schrödinger operators. The last two effects depend on the number-theoretic properties of the model parameters.

Quantum SU(3) systems: (non)integrability, quantum chaos, classical limit(s), and experiments

M. Kuś
Center for Theoretical Physics, Polish Academy of Sciences, Warsaw, Poland

Quantum systems with the SU(3) symmetry, having their origin in nuclear physics, were a fruitful playground for quantum chaos investigations, in particular due to they reach possible behavior in the classical limit. Even for the simplest models intriguing questions concerning integrability in classical limit can be posed, especially in view of possible experimental realizations of such models with trapped ions. I will present some interesting aspects of such models: various classical limits, algebraic (non)integrability, and implementations.

Tuning of Fano resonances by rotation of continuum: wave faucet

A. F. Sadreev
Kirensky Institute of Physics, Krasnoyarsk, Russia

Typically the Fano resonance asymmetric shape is tuned by reconstruction of eigenvalue spectrum by, for example, application of finger gate potential in quantum dots or magnetic field in rings. But in fact the resonances of open quantum system are given by poles of the S-matrix or the complex eigenvalues of the effective non Hermitian Hamiltonian [1] $\widehat{H}_{eff} = \widehat{H}_R - i\widehat{W}\widehat{W}^\dagger$ where $\widehat{H}_R$ describes the closed system (resonator) with discrete eigenvalue spectrum and $\widehat{W}$ describes the coupling matrix between the closed system and the continua (of attached waveguides) [2]. Typically the real parts of complex eigenvalues of the effective Hamiltonian which define the resonance positions follow the eigenvalues of the closed resonator when the coupling is weak. However with growth of the coupling the resonances become to interfere to give rise to tune the Fano resonance profiles. First this way to tune the Fano resonance was demonstrated by Rotter *et al* [3] by implementation of changeable diaphragms at the waveguide junctions.

We consider a system which changes only a phase of the coupling matrix by rotation of one of waveguides and show that gives rise to drastic effects beginning with opening and closing the wave transmission (wave faucet) and ending by the bound states in the continuum (BSC). Rotation of one of the waveguide relative to another can be performed in a realistic acoustic or electromagnetic experiment by the use of piston-like hollow-stem waveguides tightly fit to the interior boundaries of a cylindric cavity. The most striking effect of the rotation is a collapse of the Fano resonances that evidences an occurrence of bound states in the continuum (BSCs) when the resonator traps the propagating mode inside. The last has become one of the actively studied phenomena in different areas of physics [4].

References

[1] H. Feshbach H. Ann. Phys. N.Y. **5**, 357 (1958).

[2] K. Pichugin, H. Schanz, and P. Seba, Phys. Rev. E**64**, 056227 (2001).

[3] S. Rotter, F. Libisch, J. Burgoffer, U. Kuhl, and H.-J. Stöckmann, Phys. Rev. E **69**, 046208 (2004).

[4] Chia Wei Hsu, Bo Zhen, A.D. Stone, J.D. Joannopoulos, and M. Soljačić, Nature Rev. Mat. **1**, 16048 (2016).

Missing level statistics of microwave networks with violated time reversal invariance

L. Sirko
Institute of Physics, Polish Academy of Sciences, Poland

In the talk I will discuss experimental studies of the power spectrum and the other fluctuation properties of the spectra of microwave networks simulating chaotic quantum graphs with broken time reversal symmetry. In the measurements a few percent of the levels were missing in each realization of a microwave network.

On the basis of our data sets we demonstrate that the power spectrum in combination with long-range and short-range level correlations provides a powerful tool for the identification of the symmetries and the determination of the fraction of missing levels [1]. It is important to point out that such a procedure is indispensable for the evaluation of the fluctuation properties in the spectra of, e.g., molecules or nuclei, where one has to deal with missing levels.

This work was partially supported by the Ministry of Science and Higher Education grant UMO-2013/09/D/ST2/03727 and the EAgLE project (FP7-REGPOT-2013-1, Project Number: 316014).

References

[1]. M. Białous, V. Yunko, S. Bauch, M. Ławniczak, B. Dietz, and L. Sirko, *Phys. Rev. Lett.* **117**, 144101 (2016).

A microwave realization of the Gaussian symplectic ensemble

H.-J. Stöckmann
Universität Marburg, Marburg, Germany

Following an idea by Joyner et al. [1] a microwave graph with an antiunitary symmetry T obeying $T^2 = -1$ has been realized [2]. The Kramers doublets expected for such systems have been clearly identified and could be lifted by a perturbation which breaks the antiunitary symmetry. The observed spectral level spacings distribution of the Kramers doublets is in agreement with the predictions from the Gaussian symplectic ensemble (GSE), expected for chaotic systems with such a symmetry. After 50 year of random matrix theory this has been the first experimental realization of the GSE. In addition recent results on the two-point correlation function, the spectral form factor, the number variance and the spectral rigidity will be presented, as well as the transition from GSE to GOE statistics by continuously changing T from $T^2 = -1$ to $T^2 = 1$.

References

[1] C. H. Joyner, S. Müller, and M. Sieber. Europhys. Lett. 107, 50004 (2014).

[2] A. Rehemanjiang, M. Allgaier, C. H. Joyner, S. Müller, M. Sieber, U. Kuhl, and H.-J. Stöckmann. Phys. Rev. Lett. 117, 064101 (2001).

Quantum chaos in composite systems

K. Życzkowski
Jagiellonian University, Cracow, Poland and Center for Theoretical Physics, Warsaw, Poland

Unitary evolution operator of a quantum analogue of a classically chaotic system transforms a typical initial state into a delocalized random pure state. Analyzing such a unitary dynamics for a composite, bipartite system and performing partial trace over a selected subsystem one obtains a generic mixed state on the second subsystem. We investigate statistical properties of such generic mixed states and show that for a large dimension of the Hilbert space they become universal due to the effect of concentration of measure. In particular the trace distance between two random mixed states converges to $1/2 + 2/\pi$, which due to the Helstom bound determines their discriminarion in an optimal measurement scheme.

Bibliography

[1] Z. Puchała, Ł. Pawela, K. Życzkowski, Distinguishability of generic quantum states, *Phys. Rev.* **A 93**, 061221 (2016).

Contributed talks

Semiclassical bounds on Magnetic Laplacian

D. Barseghyan
University of Ostrava, Ostrava, Czechia

We derive spectral estimates into several classes of magnetic systems. They include three-dimensional regions with Dirichlet boundary as well as a particle in $\mathbb{R}^3$ confined by a local change of the magnetic field. We establish two-dimensional Berezin-Li-Yau and Lieb-Thirring-type bounds in the presence of magnetic fields and, using them, get three-dimensional estimates for the eigenvalue moments of the corresponding magnetic Laplacians.

The Brownian traveller on manifolds

D. Krejčiřík
Faculty of Nuclear Sciences and Physical Engineering, Czech Technical University in Prague, Prague, Czechia

One of the most influential works of Petr Šeba's is about geometrically induced bound states in quantum waveguides. Mathematically, and more generally, the problem is concerned with the spectral analysis of the Laplacian in tubular neighbourhoods of submanifolds of Riemannian manifolds. Using a stochastic motivation, we make an overview of our contribution on variants of such problems related to the Petr's pioneering work.

Chaotic behaviour of the electricity price series

R. Kříž
University of Hradec Králové, Hradec Králové, Czechia

Electricity price is by its features like mean-reversion, high volatility rate and frequent occurrence of jumps different from other commodities. These differences are mainly caused by non-storability of the electricity, which need to balance supply and demand in real time. Due to these features, electricity price behavior seems somewhat chaotic. In this paper we will introduce methods for investigating whether or not electricity spot prices can be described by usual time series, stochastic models. Classical methods as well as the latest methods, often applied for the first time to economic time series, were used. Among traditional descriptors belong mutual information, the largest Lyapunovv exponent, fractal dimension, entropy and the Hurst exponent. A fairly new method called the 0–1 chaos test was applied to detect chaotic behavior. We will do a case study on the EPEX Phelix spot index.

Fermi's rule and high-energy asymptotics for quantum graphs

J. Lipovský
University of Hradec Králové, Hradec Králové, Czechia

We investigate resonances in non-compact quantum graphs with general coupling conditions. Inspired by the recent result of Lee and Zworski for graphs with standard coupling, we derive Fermi's golden rule for general graphs, i.e. the behaviour of the second derivative of the square root of energy k with respect to the parameter giving length of the edges of the graph in the vicinity of the former eigenvalue of the graph. We approximate the trajectories of the resonances by parabolas. Furthermore, we give high-energy asymptotics of the resonances for δ and δ'_{s}-coupling. We prove that if there is δ'_{s}-coupling at all the vertices where the half-lines are attached, the resonances approach to the real axis with $\mathrm{Re}\, k \to \infty$. To obtain all the results, we use the method of pseudo-orbit expansion of the resonance condition.

This is a joint work with prof. Pavel Exner.

References

[1] Pseudo-orbit approach to trajectories of resonances in quantum graphs with general vertex coupling: Fermi rule and high-energy asymptotics, *J. Math. Phys.* **58** (2017), 042101.

Optimization of the lowest eigenvalue for surface $\boldsymbol{\delta}$-interactions

V. Lotoreichik
Nuclear Physics Institute of the Czech Academy of Sciences, Řež, Czechia

The celebrated Faber-Krahn inequality yields that, among all domains of a fixed volume, the ball minimizes the lowest eigenvalue of the Dirichlet Laplacian. This result can be viewed as a spectral counterpart of the well known geometric isoperimetric inequality. The aim of this talk is to discuss generalizations of the Faber-Krahn inequality for optimization of the lowest eigenvalue for Schrödinger operators with δ-interactions supported on surfaces. We will mainly focus on three geometric settings:

- open arcs of varying shape, under the constraint of fixed length [3];
- unbounded conical surfaces with the cross-sections of varying shape and fixed length [1];
- star-graphs with infinite leads and varying angles between them [2].

Beyond a physical relevance of δ-interactions, a purely mathematical motivation to consider these optimization problems stems from the fact that standard methods, going back to the papers of Faber and Krahn, are not applicable anymore. Our main tool in the proofs is the Birman-Schwinger principle, which we combine with various purely geometric inequalities.

References

[1] P. Exner and V. Lotoreichik, A spectral isoperimetric inequality for cones, *Lett. Math. Phys.* **107** (2017), 717–732.

[2] P. Exner and V. Lotoreichik, Optimization of the lowest eigenvalue for leaky star graphs, *submitted*, `arXiv:1701.06840`.

[3] V. Lotoreichik, Spectral isoperimetric inequalities for δ-interactions on open arcs and for the Robin Laplacian on planes with slits, *submitted*, `arXiv:1609.07598`.

On the bound states of magnetic Laplacians on wedges

A. Pérez-Obiol
Nuclear Physics Institute of the Czech Academy of Sciences, Řež, Czechia

This note is mainly inspired by the conjecture about the existence of bound states for magnetic Neumann Laplacians on planar wedges of any aperture $\phi \in (0, \pi)$. So far, a proof was only obtained for the apertures $\phi \leq \pi/2$. The conviction in the validity of this conjecture for the apertures $\phi \in (\pi/2, \pi)$ mainly relied on numerical computations. In this note we succeed to prove existence of bound states for any aperture $\phi \lesssim 0.509\pi$ using a variational argument with suitably chosen test functions. Employing some more involved test functions and combining a variational argument with numerical optimisation, we extend this interval up to any aperture $\phi \lesssim 0.595\pi$. Moreover, we analyse the same question for closely related problems concerning magnetic Robin Laplacians on wedges and for magnetic Schrödinger operators in the plane with δ-interactions supported on broken lines.

Networks as Coherent Perfect Absorbers

H. Schanz
Magdeburg-Stendal University of Applied Sciences, Magdeburg, Germany

Absorbers in systems with coherent wave propagation always introduce some back scattering which prevents the complete absorption of incident energy. However, in systems with multiple scattering and precise control over the incoming wave, coherent perfect absorption (CPA) can be achieved for specific wave numbers. We study this phenomenon in wave chaotic systems and develop a statistical description for CPA based on random matrix and perturbation theory. As a concrete numerical example with a potential experimental realizations we study quantum graphs where the absorption is concentrated in some of the vertices of the network.

References

H. Li et al.: Phys. Rev. Lett. 118 (2017) 044101.

Integrable partial differential systems in four independent variables related to contact geometry

A. Sergyeyev
Silesian University in Opava, Opava, Czechia

In this talk we present a broad new class of integrable first-order quasilinear systems in four independent variables whose Lax pairs involve contact vector fields. For further details please see arXiv:1401.2122.

Amphibious Complex Orbits and Dynamical Tunneling

A. Shudo
Tokyo Metropolitan University, Tokyo, Japan

Quantum tunneling takes place dynamically in mixed-type phase space where invariant tori and chaotic regions coexist. Complex semiclassical analysis reveals that a bunch of tunneling trajectories, not like the instanton in 1-dimensional tunneling, associated with chaos, or more precisely the Julia set in the complex plane, is involved in dynamical tunneling in mixed phase space. An important characteristic of complex orbits, which has been shown in a series of works by Bedford and Smillie, is that they exhibit an amphibious character: they behave as regular orbits when the orbits stay in the regular region while as chaotic orbits when they wander in the chaotic domain. Exactly this character explains the emergence of "amphibious states", the states ignoring the underlying classical invariant structures.

Ionospehere as chaotic billiard and what it can reveal

J. Šlégr
University of Hradec Králové, Hradec Králové, Czechia

Microwave billiards are currently under study as an example of dynamical billiards. In this contribution we show that ionopshere has in many ways analogous properties. Ionosphere acts as Gaussian random plane for the reflection of very low frequency waves.

Chaos in the (feline) brain

F. Studnička
University of Hradec Králové, Hradec Králové, Czechia

Chaotic patterns in the brain emerge as stereotypes in the behavior of (not only) human beings. In the first part, some interesting correlation between terrorist attacks and newspaper articles will be presented as a simple mathematical model of rumor spreading. In the second part, it will be proven, why cats cannot survive fall from the seventh floor, but are likely to survive fall from higher floors. The coefficient of cat's fear will be derived in order to understand what is happening in the feline brain during the free fall.

Spectral and resonance properties of the Smilansky model

M. Tater
Nuclear Physics Institute of the Czech Academy of Sciences, Řež, Czechia

We analyze the Hamiltonian proposed by Smilansky to describe irreversible dynamics in quantum graphs and studied further by Solomyak and others. We derive a weak-coupling asymptotics of the ground state and add new insights by finding the discrete spectrum numerically in the subcritical case. Furthermore, we show that the model then has a rich resonance structure.

This is a joint work with P. Exner and V. Lotoreichik.

References

[1] P. Exner, V. Lotoreichik, M. Tater, Spectral and resonance properties of Smilansky Hamiltonian, *Phys. Lett. A*\textbf{381} (2017), 756-761

[2] P. Exner, V. Lotoreichik, M. Tater, On resonances and bound states of Smilansky Hamiltonian, *Nanosystems: Phys. Chem. Math.* **7** (2016), 789-802

Heisenberg picture in non-Hermitian quantum mechanics

M. Znojil
Nuclear Physics Institute of the Czech Academy of Sciences, Řež, Czechia

Within a broader research subject of study of quantum unitary dynamics in non-Hermitian representation the currently most popular non-Hermitian version of Schroedinger picture is shown complemented by its Dirac's interaction picture and Heisenberg picture innovative analogues.

Social programme

Concert

The organ concert will take place on *Tuesday, May 9 at 20 pm* at the *Building C of the University of Hradec Králové*, nám. Svobody 301/1, Hradec Králové. There will be music from baroque to romanticism period on the programme. The price is included in the conference fee.

Conference dinner

The conference dinner will be held on *Wednesday, May 10 at 19:30 pm* in the *Hotel Tereziánský dvůr*, J. Koziny 336, Hradec Králové. The price is included in the conference fee.

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