Some unusual spectra of periodic quantum graphs

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The topic of quantum graphs which we opened with Petr thirty years ago appeared to be unexpectedly fruitful. In this talk I am going to discuss spectra of periodic quantum graphs using two simple examples, a chain of loops and a rectangular lattice. It is generally accepted that, with the exception of 'Dirichlet eigemvalues', the spectrum consists of absolutely continuous bands and that local perturbations can give rise to eigenvalues in the gaps. My goal is to show that one can observe also other types of spectral behaviour: (i) a chain in a homogeneous magnetic field can have no absolutely continuous spectrum at all, (ii) a chain in a linear magnetic can have a spectrum of a fractal nature, and (iii) even without any external field a lattice can have a finite number of spectral gaps in analogy with Bethe-Sommerfeld behaviour of the 'usual' Schrödinger operators. The last two effects depend on the number-theoretic properties of the model parameters.