Optimization of the lowest eigenvalue for surface δ-interactions

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The celebrated Faber-Krahn inequality yields that, among all domains of a fixed volume, the ball minimizes the lowest eigenvalue of the Dirichlet Laplacian. This result can be viewed as a spectral counterpart of the well known geometric isoperimetric inequality. The aim of this talk is to discuss generalizations of the Faber-Krahn inequality for optimization of the lowest eigenvalue for Schrödinger operators with δ-interactions supported on surfaces. We will mainly focus on three geometric settings:

- open arcs of varying shape, under the constraint of fixed length [3];
- unbounded conical surfaces with the cross-sections of varying shape and fixed length [1];
- star-graphs with infinite leads and varying angles between them [2].

Beyond a physical relevance of δ-interactions, a purely mathematical motivation to consider these optimization problems stems from the fact that standard methods, going back to the papers of Faber and Krahn, are not applicable anymore. Our main tool in the proofs is the Birman-Schwinger principle, which we combine with various purely geometric inequalities.

References

[1] P. Exner and V. Lotoreichik, A spectral isoperimetric inequality for cones, *Lett. Math. Phys.* **107** (2017), 717–732.

[2] P. Exner and V. Lotoreichik, Optimization of the lowest eigenvalue for leaky star graphs, *submitted*, `arXiv:1701.06840`.

[3] V. Lotoreichik, Spectral isoperimetric inequalities for δ-interactions on open arcs and for the Robin Laplacian on planes with slits, *submitted*, `arXiv:1609.07598`.